Trigonometria II

Cosseno e Seno da Soma

2° ano E.M.

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PORTAL DA MATEMÁTICA
OBMEP

Trigonometria II
Cosseno e Seno da Soma

1 Exercícios Introdutórios

Exercício 1. Se $\operatorname{sen}(a+b) = \operatorname{sen} a \cos b + \operatorname{sen} b \cos a$, para dois arcos a e b, então qual o $\operatorname{sen}(2a)$?

a) $\operatorname{sen} a \cos a$.

b) $2 \operatorname{sen} a \cos a$.

c) $\dfrac{\operatorname{sen} a \cos a}{2}$.

d) $\operatorname{sen} a + \cos a$.

e) $\operatorname{sen} a - \cos a$.

Exercício 2. Sejam dois ângulos a e b, tais que $\operatorname{sen} a = \dfrac{3}{5}$, $\cos a = \dfrac{4}{5}$, $\operatorname{sen} b = \dfrac{5}{13}$ e $\cos b = \dfrac{12}{13}$. Quanto vale $\operatorname{sen}(a+b)$?

a) $\dfrac{56}{65}$.

b) $\dfrac{57}{65}$.

c) $\dfrac{58}{65}$.

d) $\dfrac{59}{65}$.

e) $\dfrac{61}{65}$.

Exercício 3. $\operatorname{sen} 15^o$ é igual a:

a) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$.

b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$.

c) $\dfrac{\sqrt{6}+\sqrt{2}}{2}$.

d) $\dfrac{\sqrt{6}-\sqrt{2}}{2}$.

e) $\dfrac{\sqrt{3}+\sqrt{2}}{4}$.

Exercício 4. Quanto vale $\cos 15^o \cos 45^o - \operatorname{sen} 15^o \operatorname{sen} 45^o$?

a) $-\dfrac{1}{2}$.

b) $\dfrac{1}{2}$.

c) $-\dfrac{\sqrt{3}}{2}$.

d) $\dfrac{\sqrt{3}}{2}$.

e) $\dfrac{\sqrt{2}}{2}$.

Exercício 5. Se $\operatorname{tg} a = 2$, então $\operatorname{tg}(2a)$ é:

a) $\dfrac{4}{3}$.

b) $\dfrac{3}{4}$.

c) $-\dfrac{3}{4}$.

d) $-\dfrac{4}{3}$.

e) $\dfrac{2}{5}$.

Exercício 6. Calcule $\cos\left(\dfrac{\pi}{12}\right)$ e $\operatorname{tg}\left(\dfrac{\pi}{12}\right)$.

2 Exercícios de Fixação

Exercício 7. Determine $\cos 105^o$.

Exercício 8. Sejam dois arcos x e $2x$, ambos do 1^o quadrante, tais que $\operatorname{sen}(2x) = \dfrac{1}{3}$. Determine $\operatorname{sen} x$.

Exercício 9. Se $\operatorname{sen} x + \cos x = \dfrac{5}{6}$, então $\operatorname{sen}(2x)$ é igual a:

a) $-\dfrac{7}{36}$.

b) $-\dfrac{11}{36}$.

c) $-\dfrac{13}{36}$.

d) $-\dfrac{17}{36}$.

e) $-\dfrac{19}{36}$.

Exercício 10. Calcule $\operatorname{sen} 10^o \cdot \operatorname{sen} 50^o \cdot \operatorname{sen} 70^o$.

a) $-\dfrac{5}{13}$.

b) $-\dfrac{5}{12}$.

c) $\dfrac{5}{13}$.

d) $\dfrac{5}{12}$.

e) $0,334$.

Exercício 11. Calcule

$$\cos 36^o - \cos 72^o.$$

Exercício 12. Seja $\dfrac{\sqrt{3}-1}{\operatorname{sen} x} + \dfrac{\sqrt{3}+1}{\cos x} = 4\sqrt{2}$, onde $x \in [0, 360^o]$, determine x.

Exercício 13. Calcule $\cos^4\left(\dfrac{\pi}{24}\right) - \operatorname{sen}^4\left(\dfrac{\pi}{24}\right)$.

3 Exercícios de Aprofundamento e de Exames

Exercício 14. O valor de $\cos 735^o$ é:

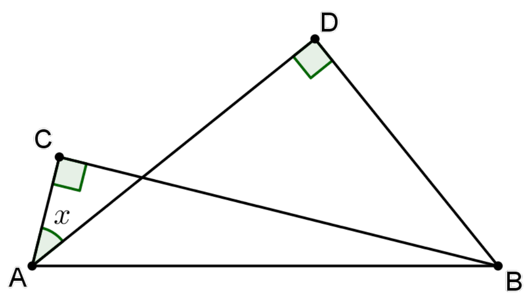
a) $\frac{1}{4}$.

b) $\frac{\sqrt{3}}{4}$.

c) $\frac{\sqrt{2}+\sqrt{6}}{4}$.

d) $\frac{\sqrt{2}-\sqrt{6}}{4}$.

Exercício 15. Nos triângulos retângulos da figura, $AC = 1$ cm, $BC = 7$ cm, $AD = BD$. Sabendo que $\text{sen}(a-b) = \text{sen}\, a \cos b - \cos a\, \text{sen}\, b$, o valor de $\text{sen}\, x$ é:



a) $\frac{\sqrt{2}}{2}$.

b) $\frac{7}{\sqrt{50}}$.

c) $\frac{3}{5}$.

d) $\frac{4}{5}$.

e) $\frac{1}{\sqrt{50}}$.

Exercício 16. Sabendo que $\pi < x < \frac{3\pi}{2}$ e $\text{sen}\, x = -\frac{1}{3}$, é correto afirmar que $\text{sen}(2x)$ é:

a) $-\frac{2}{3}$.

b) $-\frac{1}{6}$.

c) $\frac{\sqrt{3}}{8}$.

d) $\frac{1}{27}$.

e) $\frac{4\sqrt{2}}{9}$.

Exercício 17. Se $\text{sen}\, x = -\frac{2}{3}$, $\cos(2x) \cdot \text{sen}(-x)$ é:

a) $\frac{2}{9}$.

b) $\frac{2}{27}$.

c) $-\frac{2}{9}$.

d) $-\frac{2}{27}$.

e) $-\frac{9}{27}$.

Exercício 18. Os lados a, b e c de um triângulo estão em progressão aritmética, nesta ordem, sendo opostos aos ângulos internos α, β e θ, respectivamente. Determine o valor da expressão:

$$\frac{\cos \frac{\alpha-\theta}{2}}{\cos \frac{\alpha+\theta}{2}}$$

a) $\sqrt{2}$.

b) 2.

c) $2\sqrt{2}$.

d) 3.

e) 4.

Exercício 19. Seja n um inteiro positivo tal que $\text{sen}\, \frac{\pi}{2n} = \sqrt{\frac{2-\sqrt{3}}{4}}$.

a) Determine n.

b) Determine $\text{sen}\, \frac{\pi}{24}$.

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Respostas e Soluções.

1. B.

2.

$$\begin{aligned}
\operatorname{sen}(a+b) &= \operatorname{sen} a \cos b + \operatorname{sen} b \cos a \\
&= \frac{3}{5}\cdot\frac{12}{13} + \frac{5}{13}\cdot\frac{4}{5} \\
&= \frac{36}{65} + \frac{20}{65} \\
&= \frac{56}{65}.
\end{aligned}$$

Resposta A.

3.

$$\begin{aligned}
\operatorname{sen} 15^o &= \operatorname{sen}(45^o - 30^o) \\
&= \operatorname{sen} 45^o \cos 30^o - \operatorname{sen} 30^o \cos 45^o \\
&= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} \\
&= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{6}-\sqrt{2}}{4}.
\end{aligned}$$

Resposta B.

4. $\cos 15^o \cos 45^o - \operatorname{sen} 15^o \operatorname{sen} 45^o = \cos(15^o + 45^o) = \cos 60^o = \frac{1}{2}$. Resposta B.

5.

$$\begin{aligned}
\operatorname{tg}(2a) &= \frac{2\operatorname{tg} a}{1-(\operatorname{tg} a)^2} \\
&= \frac{2\cdot 2}{1-2^2} \\
&= \frac{4}{-3} \\
&= -\frac{4}{3}.
\end{aligned}$$

Resposta D.

6. (Extraído da Vídeo Aula)

$$\begin{aligned}
\cos\frac{\pi}{12} &= \cos 15^o \\
&= \cos(45^o - 30^o) \\
&= \cos 45^o \cdot \cos 30^o + \operatorname{sen} 45^o \cdot \operatorname{sen} 30^o \\
&= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} \\
&= \frac{\sqrt{6}+\sqrt{2}}{4}.
\end{aligned}$$

$$\begin{aligned}
\operatorname{tg}\frac{\pi}{12} &= \operatorname{tg} 15^o \\
&= \operatorname{tg}(45^o - 30^o) \\
&= \frac{\operatorname{tg} 45^o - \operatorname{tg} 30^o}{1+\operatorname{tg} 45^o \cdot \operatorname{tg} 30^o} \\
&= \frac{1-\frac{\sqrt{3}}{3}}{1+1\cdot\frac{\sqrt{3}}{3}} \\
&= \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} \\
&= \frac{3-\sqrt{3}}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}} \\
&= \frac{9-6\sqrt{3}+3}{9-3} \\
&= 2-\sqrt{3}.
\end{aligned}$$

7.

$$\begin{aligned}
\cos 105^o &= \cos(60^o + 45^o) \\
&= \cos 60^o \cdot \cos 45^o - \operatorname{sen} 60^o \cdot \operatorname{sen} 45^o \\
&= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} \\
&= \frac{\sqrt{2}-\sqrt{6}}{4}.
\end{aligned}$$

8. Se $\operatorname{sen}(2x) = \frac{1}{3}$, então $\cos(2x) = \frac{2\sqrt{2}}{3}$. Temos, assim:

$$\begin{aligned}
\cos(2x) &= \frac{2\sqrt{2}}{3} \\
(\cos x)^2 - (\operatorname{sen} x)^2 &= \frac{2\sqrt{2}}{3} \\
1-(\operatorname{sen} x)^2 - (\operatorname{sen} x)^2 &= \frac{2\sqrt{2}}{3} \\
2(\operatorname{sen} x)^2 &= 1-\frac{2\sqrt{2}}{3} \\
\operatorname{sen} x &= \sqrt{\frac{3-2\sqrt{2}}{6}}.
\end{aligned}$$

9.

$$\begin{aligned}
(\operatorname{sen} x + \cos x)^2 &= \left(\frac{5}{6}\right)^2 \\
(\operatorname{sen} x)^2 + 2\operatorname{sen} x \cos x + (\cos x)^2 &= \frac{25}{36} \\
1+\operatorname{sen}(2x) &= \frac{25}{36} \\
\operatorname{sen}(2x) &= -\frac{11}{36}.
\end{aligned}$$

10. (Extraído da Vídeo Aula)

$$
\begin{aligned}
\operatorname{sen} 10^o \cdot \operatorname{sen} 50^o \cdot \operatorname{sen} 70^o &= \\
\frac{2 \operatorname{sen} 10^o \cdot \cos 10^o \cdot \operatorname{sen} 50^o \cdot \operatorname{sen} 70^o}{2 \cos 10^o} &= \\
\frac{\operatorname{sen}(2 \cdot 10^o) \cdot \operatorname{sen} 50^o \cdot \operatorname{sen} 70^o}{2 \cos 10^o} &= \\
\frac{\operatorname{sen} 20^o \cdot \operatorname{sen} 50^o \cdot \cos 20^o}{2 \cos 10^o} &= \\
\frac{2 \operatorname{sen} 20^o \cdot \cos 20^o \cdot \operatorname{sen} 50^o}{2 \cdot 2 \cdot \cos 10^o} &= \\
\frac{\operatorname{sen} 40^o \cdot \cos 40^o}{4 \cos 10^o} &= \\
\frac{\operatorname{sen} 80^o}{8 \cos 10^o} &= \\
\frac{\cos 10^o}{8 \cos 10^o} &= \frac{1}{8}.
\end{aligned}
$$

11. (Extraído da Vídeo Aula)

$$
\begin{aligned}
\cos 36^o - \cos 72^o &= \\
\frac{(\cos 36^o - \cos 72^o) \cdot 2(\cos 36^o + \cos 72^o)}{2(\cos 36^o + \cos 72^o)} &= \\
\frac{2((\cos 36^o)^2 - (\cos 72^o)^2)}{2(\cos 36^o + \cos 72^o)} &= \\
\frac{2\left(\frac{1+\cos 72^o}{2} - \frac{1+\cos 144^o}{2}\right)}{2(\cos 36^o + \cos 72^o)} &= \\
\frac{\cos 72^o - \cos 144^o}{2(\cos 36^o + \cos 72^o)} &= \\
\frac{\cos 72^o + \cos 36^o}{2(\cos 36^o + \cos 72^o)} &= \frac{1}{2}.
\end{aligned}
$$

12. (Extraído da Vídeo Aula)

$$
\begin{aligned}
\frac{\sqrt{3}-1}{\operatorname{sen} x} + \frac{\sqrt{3}+1}{\cos x} &= 4\sqrt{2} \\
\frac{\frac{\sqrt{2}}{4}(\sqrt{3}-1)}{\operatorname{sen} x} + \frac{\frac{\sqrt{2}}{4}(\sqrt{3}+1)}{\cos x} &= \frac{\sqrt{2}}{4} \cdot 4\sqrt{2} \\
\frac{\operatorname{sen} 15^o}{\operatorname{sen} x} + \frac{\cos 15^o}{\cos x} &= 2 \\
\frac{\operatorname{sen} 15^o \cos x}{\operatorname{sen} x \cos x} + \frac{\cos 15^o \operatorname{sen} x}{\cos x \operatorname{sen} x} &= 2 \\
\frac{2 \operatorname{sen}(x + 15^o)}{\operatorname{sen}(2x)} &= 2 \\
\operatorname{sen}(x + 15^o) &= \operatorname{sen}(x + x) \\
x_1 &= 15^o \\
x_2 &= 55^o \\
x_3 &= 175^o \\
x_4 &= 295^o.
\end{aligned}
$$

13. (Extraído da Vídeo Aula)

$$
\begin{aligned}
\left(\cos \frac{\pi}{24}\right)^4 - \left(\operatorname{sen} \frac{\pi}{24}\right)^4 &= \\
\left[\left(\cos \frac{\pi}{24}\right)^2 + \left(\operatorname{sen} \frac{\pi}{24}\right)^2\right]\left[\left(\cos \frac{\pi}{24}\right)^2 - \left(\operatorname{sen} \frac{\pi}{24}\right)^2\right] &= \\
1 \cdot \left[\cos\left(2 \cdot \frac{\pi}{24}\right)\right] &= \\
\cos \frac{\pi}{12} &= \\
\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) &= \\
\cos \frac{\pi}{4} \cos \frac{\pi}{6} + \operatorname{sen} \frac{\pi}{4} \operatorname{sen} \frac{\pi}{6} &= \\
\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} &= \\
\frac{\sqrt{6}+\sqrt{2}}{2}.
\end{aligned}
$$

14. (Extraído da EEAR - 2016)

$$
\begin{aligned}
\cos 735^o &= \cos 15^o \\
&= \cos(45^o - 30^o) \\
&= \cos 45^o \cos 30^o + \operatorname{sen} 45^o \operatorname{sen} 30^o \\
&= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\
&= \frac{\sqrt{6}+\sqrt{2}}{4}.
\end{aligned}
$$

Resposta C.

15. (Extraído da FUVEST - SP) Usando o Teorema de Pitágoras, temos $AB = 5\sqrt{2}$ e, consequentemente, $AD = BD = 5$. Assim:

$$
\begin{aligned}
\operatorname{sen} x &= \operatorname{sen}(\angle BAC - \angle BAD) \\
&= \operatorname{sen} \angle BAC \cdot \cos \angle BAD - \operatorname{sen} \angle BAD \cdot \cos \angle BAC \\
&= \frac{7}{5\sqrt{2}} \cdot \frac{5}{5\sqrt{2}} - \frac{5}{5\sqrt{2}} \cdot \frac{1}{5\sqrt{2}} \\
&= \frac{7}{10} - \frac{1}{10} \\
&= \frac{3}{5}.
\end{aligned}
$$

Resposta C.

16. (Extraído da PUC - RJ - 2015) Se $\operatorname{sen} x = -\frac{1}{3}$, então $\cos = -\frac{2\sqrt{2}}{3}$. Assim, temos $\operatorname{sen}(2x) = 2 \operatorname{sen} x \cos x = 2\left(-\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right) = \frac{4\sqrt{2}}{9}$. Resposta E.

17. (Extraído da IFCE - 2014) Se $\operatorname{sen} x = -\frac{2}{3}$, então $\cos x$ pode ser $\frac{\sqrt{5}}{3}$ ou $-\frac{\sqrt{5}}{3}$. Além disso, $\operatorname{sen}(-x) = -\operatorname{sen} x = \frac{2}{3}$.

Sendo assim, temos:

$$\begin{aligned}\cos(2x)\cdot\operatorname{sen}(-x) &= \\ ((\cos x)^2-(\operatorname{sen} x)^2)\cdot\frac{2}{3} &= \\ \left(\frac{5}{9}-\frac{4}{9}\right)\cdot\frac{2}{3} &= \\ \frac{1}{9}\cdot\frac{2}{3} &= \frac{2}{27}.\end{aligned}$$

Resposta B.

18. (Extraído do IME - 2015) Seja r a razão da PA, então $a=b-r$ e $c=b+r$. Aplicando a Lei dos Cossenos, temos: $\cos\alpha=\dfrac{b^2+c^2-a^2}{2bc}=\dfrac{b^2+(b+r)^2-(b-r)^2}{2b(b+r)}=\dfrac{b+4r}{2(b+r)}$ e $\cos\theta=\dfrac{a^2+b^2-c^2}{2bc}=\dfrac{(b-r)^2+b^2-(b+r)^2}{2(b-r)b}=\dfrac{b-4r}{2(b-r)}$.

Temos ainda que $\operatorname{tg}\dfrac{\alpha}{2}=\sqrt{\dfrac{1-\dfrac{b+4r}{2(b+r)}}{1+\dfrac{b+4r}{2(b+r)}}}=\sqrt{\dfrac{b-2r}{3(b+2r)}}$ e $\operatorname{tg}\dfrac{\theta}{2}=\sqrt{\dfrac{1-\dfrac{b-4r}{2(b-r)}}{1+\dfrac{b-4r}{2(b-r)}}}=\sqrt{\dfrac{b+2r}{3(b-2r)}}$. Multiplicando os valores encontrados para estas tangentes, encontramos $\operatorname{tg}\dfrac{\alpha}{2}\cdot\operatorname{tg}\dfrac{\theta}{2}=\sqrt{\dfrac{(b-2r)(b+2r)}{9(b+2r)(b-2r)}}=\dfrac{1}{3}$. Por fim, temos:

$$\begin{aligned}\frac{\cos\dfrac{\alpha-\theta}{2}}{\cos\dfrac{\alpha+\theta}{2}} &= \\ \frac{\cos\dfrac{\alpha}{2}\cos\dfrac{\theta}{2}+\operatorname{sen}\dfrac{\alpha}{2}\operatorname{sen}\dfrac{\theta}{2}}{\cos\dfrac{\alpha}{2}\cos\dfrac{\theta}{2}-\operatorname{sen}\dfrac{\alpha}{2}\operatorname{sen}\dfrac{\theta}{2}} &= \\ \frac{1+\operatorname{tg}\dfrac{\alpha}{2}\operatorname{tg}\dfrac{\theta}{2}}{1-\operatorname{tg}\dfrac{\alpha}{2}\operatorname{tg}\dfrac{\theta}{2}} &= \\ \frac{1+\dfrac{1}{3}}{1-\dfrac{1}{3}} &= 2.\end{aligned}$$

Resposta B.

19. (Extraído do ITA - 2015)

a)

$$\begin{aligned}\operatorname{sen}\frac{\pi}{2n} &= \sqrt{\frac{2-\sqrt{3}}{4}} \\ \left(\operatorname{sen}\frac{\pi}{2n}\right)^2 &= \frac{2-\sqrt{3}}{4} \\ \sqrt{3} &= 2-4\left(\operatorname{sen}\frac{\pi}{2n}\right)^2 \\ \frac{\sqrt{3}}{2} &= 1-2\left(\operatorname{sen}\frac{\pi}{2n}\right)^2 \\ \frac{\sqrt{3}}{2} &= \cos\frac{\pi}{n} \\ n &= 6.\end{aligned}$$

b) Se $\operatorname{sen}\dfrac{\pi}{12}=\sqrt{\dfrac{2-\sqrt{3}}{4}}$, então $\cos\dfrac{\pi}{12}=\sqrt{\dfrac{2+\sqrt{3}}{2}}$. Além disso, se $\cos(2x)=1-2sen^2x$, então $\operatorname{sen}^2 x=\dfrac{1-cos(2x)}{2}$, sendo que, para $x=\dfrac{\pi}{24}$, temos $\operatorname{sen}^2\dfrac{\pi}{24}=\dfrac{2-\sqrt{2+\sqrt{3}}}{4}$, segue que $\operatorname{sen}\dfrac{\pi}{24}=\dfrac{\sqrt{2-\sqrt{2+\sqrt{3}}}}{2}$.

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